Gravity as Nonmetricity

General Relativity in Metric-Affine Space (L_n,g)

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Abstract

In this paper we propose a new geometric interpretation for General Relativity (GR). It has always been presumed that the gravitational field is described in GR by a Levi-Civita connection. We suggest that this may not necessarily be the case. We show that in the presence of an arbitrary affine connection, the gravitational field is described as nonmetricity of the affine connection. An affine connection can be interpreted as induced by a frame of reference (FR), in which the gravitational field is considered. This leads to some interesting observations, among which: (a) gravity is a nonmetricity of space-time; (b) the affine curvature of space-time induced in a noninertial FR contributes to the stress-energy tensor of matter as an additional source of gravity; and (c) the scalar curvature of the affine connection plays the role of a "cosmological constant". It is interesting to note that although the gravitational field equations are identical to Einstein's equations of GR, this formulation leads to a covariant tensor (instead of the pseudotensor) of energy-momentum of the gravitational field and covariant conservation laws. We further develop a geometric representation of FR as a metric-affine space, with transition between FRs represented as affine deformation of the connection. We show that the affine connection of a NIFR has curvature and may have torsion. We calculate the curvature for the uniformly accelerated FR. Finally, we show that GR is inadequate to describe the gravitational field in a NIFR. We propose a generalization of GR that describes gravity as nonmetricity of the affine connection induced in a FR. The field equations coincide with Einstein's except that all partial derivatives of the metric are replaced by covariant derivatives with respect to the affine connection. This generalization contains GR as a special case of the inertial FR.

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Introduction

In the Riemannian space V_4 of General Relativity (GR) two principal geometric objects, metric $\boldsymbol{g}$ and connection Γ, are linked through the requirement of metric homogeneity, i.e. the covariant derivative of metric vanishes identically: $\nabla g = 0$. This condition assures that the length of a vector transported parallel in any direction remains invariant. Since GR was first formulated, metric $\boldsymbol{g}$ and the Levi-Civita connection Γ have been considered respectively as a potential and strength of the gravitational field. It is easy to see that the well-known difficulties, such as non-covariance of the energy-momentum pseudo-tensor of the gravitational field, that have plagued GR are directly related to the choice of noncovariant connection Γ (which is not a tensor) as the strength of a gravitational field. We will endeavor to demonstrate in the following that this need not be the case. In part I we show that in the presence of an arbitrary affine connection, the Einstein field equations lend themselves to a novel geometrical interpretation wherein the affine deformation tensor of the Levi-Civita connection plays the role of a gravitational field. Furthermore, in the case of an affine connection with vanishing torsion, the gravitational field becomes the nonmetricity of spacetime. In this section we are not concerned with the nature of this auxiliary affine connection and can consider it as merely a convenient device. The fact that these results hold true for any auxiliary affine connection suggests that this geometric interpretation is merely a recasting of GR in a new light, which does not change the field equations or any

of the predictions of the theory. The advantage of this geometric interpretation of gravity as affine deformation or nonmetricity is that it leads to a fully covariant theory with a true tensor for energy momentum of the gravitational field.

In part II we show that, guided by the ideas of geometrodynamics, we are compelled to describe inertial forces (as well as gravity) as affine deformation or nonmetricity. In this regard, the results obtained in Part I appear not at all surprising. In this section we also offer a possible physical interpretation of an auxiliary affine connection as induced by a chosen frame of reference, in which the field is considered. This interpretation of the auxiliary affine connection as no longer optional, but rather as a required part of the description of physical reality, makes the new geometric interpretation of GR offered herein so much more compelling. We describe the geometry in a noninertial frame of reference and calculate the curvature in a uniformly accelerating FR.

In part III we consider gravity in a noninertial frame of reference. We demonstrate that GR is unsuitable for describing the gravitational field in a noninertial FR. We propose here a generalization of GR wherein the gravitational field is a nonmetricity of the affine connection induced in a chosen FR. We show here that the inertial forces play the role of a gauge field, which must be turned on to compensate for the choice of a noninertial FR. This theory contains GR as a special case of the gravitational field in an inertial frame of reference.

1   General Relativity in a Metric-Affine Space ($\mathbf{L}_n$,g)

We start with Riemannian space V_4 – the standard geometrical setup of Einstein's GR comprising a differential manifold M_4 with metric $\boldsymbol{g}$ and Levi-Civita connection Γ. Let us introduce some arbitrary affine connection $\overline{\Gamma}$ on the same manifold M_4. This auxiliary connection is in no way linked with either the metric $\boldsymbol{g}$ nor with the Levi-Civita connection Γ. Such a geometric structure is usually called a metric-affine space (L_4,g). It is important to bear in mind that at this point, the affine connection $\overline{\Gamma}$ has no particular meaning and is purely arbitrary.[*]

As is well-known in differential geometry, affine connection $\overline{\Gamma}$ on a differential manifold M with the metric $\boldsymbol{g}$ can be always decomposed into the sum of the Levi-Civita (metric) connection Γ, nonmetricity $\mathbf{S}$ and torsion $\mathbf{Q}$:

$$\overline{\Gamma} = \Gamma + S + Q. \tag{1}$$

Note that connections $\overline{\Gamma}$ and Γ are not tensors, while nonmetricity $\mathbf{S}$ and torsion $\mathbf{Q}$ are tensors. In a local chart x we can write the expression (1) in components:

$$\overline{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + S^{\lambda}_{\mu\nu} + Q^{\lambda}_{\mu\nu}. \tag{2}$$

The tensor of nonmetricity $S^{\lambda}_{\mu\nu}$ is symmetric in its two lower indices while the torsion tensor $Q^{\lambda}_{\mu\nu}$ is antisymmetric in the two lower indices. An affine connection with vanishing nonmetricity, ($\mathbf{S}$=0, $\overline{\Gamma}$=Γ+$\mathbf{Q}$), is called a Riemannian connection. A Riemannian connection without torsion, ($\mathbf{Q}$=0, $\overline{\Gamma}$=Γ), is called a Levi-Civita connection. If the affine connection has a vanishing torsion and, therefore, is symmetric, we will denote it as $\overline{\overline{\Gamma}}$. Although torsion may play an important role both in field theory and in the description of non-inertial frames

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[*] The use of an auxiliary geometric device as an aid in the study of the subject at hand is not unusual in geometry where it is common, for example, to study n-dimensional manifolds as submerged in manifolds of higher dimension. For the time being, we shall consider our auxiliary affine connection as merely an aid in the study of the geometrical characteristics of the gravitational field. The physical meaning of this connection will be clarified at a later point.

of reference, for the sake of simplicity, unless otherwise stated we shall assume vanishing torsion, $\mathbf{Q}=0$. Thus, all affine connections considered herein are symmetric.

Let $g_{\mu\nu}$ be a metric tensor; $\overline{\nabla}$ be a covariant derivative with respect to the affine connection $\bar{\Gamma}^\lambda_{\mu\nu}$. The nonmetricity tensor $S^\lambda_{\mu\nu}$ can be expressed as

$$g_{\tau\sigma}S^\tau_{\mu\nu} = \frac{1}{2}\left(\overline{\nabla}_\mu g_{\nu\sigma} + \overline{\nabla}_\nu g_{\mu\sigma} - \overline{\nabla}_\sigma g_{\mu\nu}\right) \tag{3}$$

or

$$g_{\tau\sigma}S^\tau_{\mu\nu} = \frac{1}{2}\left(\rho_{\mu\nu\sigma} + \rho_{\nu\mu\sigma} - \rho_{\sigma\mu\nu}\right), \tag{4}$$

where

$$\rho_{\mu\nu\sigma} = \overline{\nabla}_\mu g_{\nu\sigma} \tag{5}$$

is the metric inhomogeneity tensor.

For any given Levi-Civita connection Γ and affine connection $\bar{\Gamma}$, there is a unique decomposition [1] of the connection

$$\begin{aligned}\Gamma &= \bar{\Gamma} - \mathbf{S}, \\ \Gamma^\lambda_{\mu\nu} &= \bar{\Gamma}^\lambda_{\mu\nu} - S^\lambda_{\mu\nu};\end{aligned} \tag{6}$$

of the Riemann curvature tensor

$$\begin{aligned}\mathbf{R} &= \bar{\mathbf{R}} - \hat{\mathbf{R}}, \\ R^\varepsilon_{\lambda\mu\nu} &= \bar{R}^\varepsilon_{\lambda\mu\nu} - \hat{R}^\varepsilon_{\lambda\mu\nu};\end{aligned} \tag{7}$$

of the Ricci tensor

$$\begin{aligned}\mathbf{Rc} &= \bar{\mathbf{R}}\mathbf{c} - \hat{\mathbf{R}}\mathbf{c}, \\ R_{\mu\nu} &= \bar{R}_{\mu\nu} - \hat{R}_{\mu\nu};\end{aligned} \tag{8}$$

of the scalar curvature

$$R = \bar{R} - \hat{R}; \tag{9}$$

and of the Einstein tensor

$$\begin{aligned}\mathbf{G} &= \bar{\mathbf{G}} - \hat{\mathbf{G}}, \\ G_{\mu\nu} &= \bar{G}_{\mu\nu} - \hat{G}_{\mu\nu};\end{aligned} \tag{10}$$

where $\mathbf{R}$ $(R^\varepsilon_{\lambda\mu\nu})$, $\mathbf{Rc}$ $(R_{\mu\nu})$, R, and $\mathbf{G}$ $(G_{\mu\nu})$ are respectively the Riemann curvature tensor, Ricci tensor, scalar curvature, and Einstein tensor, $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, of the Levi-Civita connection Γ; $\bar{\mathbf{R}}$ $\left(\bar{R}^\varepsilon_{\lambda\mu\nu}\right)$, $\bar{\mathbf{R}}\mathbf{c}$ $\left(\bar{R}_{\mu\nu}\right)$, $\bar{R}$, and $\bar{\mathbf{G}}$ $\left(\bar{G}_{\mu\nu}\right)$ are respectively the Riemann curvature tensor, Ricci tensor, scalar curvature and Einstein tensor of the affine connection $\bar{\Gamma}$; $\hat{\mathbf{R}}$ $\left(\hat{R}^\varepsilon_{\lambda\mu\nu}\right)$, $\hat{\mathbf{R}}\mathbf{c}$ $\left(\hat{R}_{\mu\nu}\right)$, $\hat{R}$, and $\hat{\mathbf{G}}$ $\left(\hat{G}_{\mu\nu}\right)$ are nonmetric components of, respectively, the Riemann curvature tensor, Ricci tensor, scalar curvature and Einstein tensor of the affine connection $\bar{\Gamma}$ defined as follows:

$$\hat{R}^\varepsilon_{\lambda\mu\nu} = \overline{\nabla}_\lambda S^\varepsilon_{\mu\nu} - \overline{\nabla}_\mu S^\varepsilon_{\lambda\nu} + S^\varepsilon_{\lambda\tau}S^\tau_{\mu\nu} - S^\varepsilon_{\lambda\mu}S^\tau_{\nu\tau}, \tag{11}$$

wherein $\hat{R}_{\mu\nu} \equiv \hat{R}^\lambda_{\lambda\mu\nu}$, $\hat{R} \equiv \hat{R}_{\mu\nu}g^{\mu\nu}$ and

$$\hat{G}_{\mu\nu} \equiv \hat{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\hat{R}. \tag{12}$$

It may be useful to summarize these definitions in the following table:

Geometric Object	Levi-Civita Connection Γ ($\Gamma^\lambda_{\mu\nu}$)	Affine Connection $\bar\Gamma$ ($\bar\Gamma^\lambda_{\mu\nu}$)	Nonmetricity Tensor $\mathbf{S}$ ($S^\lambda_{\mu\nu}$)
Riemann Curvature Tensor	$\mathbf{R}$ ($R^\varepsilon_{\lambda\mu\nu}$)	$\bar{\mathbf{R}}\left(\bar R^\varepsilon_{\lambda\mu\nu}\right)$	$\hat{\mathbf{R}}\left(\hat R^\varepsilon_{\lambda\mu\nu}\right)$
Ricci Tensor	$\mathbf{Rc}$ ($R_{\mu\nu}$)	$\bar{\mathbf{Rc}}\left(\bar R_{\mu\nu}\right)$	$\hat{\mathbf{Rc}}\left(\hat R_{\mu\nu}\right)$
Scalar Curvature	R	$\bar R$	$\hat R$
Einstein Tensor	$\mathbf{G}$ ($G_{\mu\nu}$)	$\bar{\mathbf{G}}\left(\bar G_{\mu\nu}\right)$	$\hat{\mathbf{G}}\left(\hat G_{\mu\nu}\right)$

Let us now consider Einstein's equation for the gravitational field:

$$\mathbf{G} = 8\pi\mathbf{T},$$
$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{13}$$

We use here geometrical units wherein the speed of light constant and the gravitational constant of Newton are both set to unity. In view of (10) the expression (13) can be recast as [1]

$$\hat{\mathbf{G}} = 8\pi\bar{\mathbf{T}},$$
$$\hat G_{\mu\nu} = 8\pi\bar T_{\mu\nu}, \tag{14}$$

where the modified stress-energy tensor T is defined as

$$\bar{\mathbf{T}} = \mathbf{T} - \frac{1}{8\pi}\bar{\mathbf{G}},$$
$$\bar T_{\mu\nu} = T_{\mu\nu} - \frac{1}{8\pi}\bar G_{\mu\nu}. \tag{15}$$

It is easy to see that the tensor $\hat{\mathbf{G}}$ has vanishing covariant divergence with respect to the Levi-Civita connection Γ: $\hat G^\nu_{\mu;\nu} = 0$, i.e. the Bianchi identity holds true, and consequently the new stress-energy tensor $\hat{\mathbf{T}}$ satisfies the conservation laws: $T^\nu_{\mu;\nu} = 0$. Thus, equations (14)-(15) can serve as the equations for the gravitational field. These equations look very much like Einstein's original field equation (13). In fact, these equations are equivalent to Einstein's equations, from which they were derived. Since the unique decomposition of the Levi-Civita connection and all curvature tensors into their respective affine and nonmetric components (6)–(10) hold true for any affine connection $\bar\Gamma$, which is unrelated to the gravitational field or its source, these equations are identically equivalent to Einstein's standard equation (13) and we are still on the firm ground of classical General Relativity. And yet, these equations in the form (14)-(15) present quite a different geometrical picture. The left side of equation (14) describes the gravitational field as the nonmetricity of the chosen affine connection, which in turn contributes the stress-energy tensor T as an additional source of the gravitational field.

Let us emphasize that unlike metric-affine theories of gravitation[2], which consider the metric and connection (and, sometimes, the coframe) to be independent field potentials, this reformulation of GR still considers only the metric to be the gravitational potential. The affine connection is unrelated to the gravitational field and is not a dynamic variable. It is defined ad hoc so that its curvature tensors may be computed outside of the field equations. It is most convenient to choose a connection with zero curvature, which is just as suitable for our purposes, although any other connection may be used.

What is the physical meaning of the affine connection $\bar\Gamma$? As we shall see in the next section, this connection may represent the geometry of the frame of reference (FR), in which the gravitational field is considered. Obviously, the geometry of the FR, i.e. the affine connection $\bar\Gamma$, does not depend on the stress-energy tensor $\mathbf{T}$ and does not represent the gravitational field. Therefore, in [1], we moved the affine Einstein tensor $\bar{\mathbf{G}}$ to the right side of the equation, which also reflects the fact that the inertial forces generated in a noninertial frame of reference have energy and, therefore, contribute to the stress-energy

tensor of matter as an additional source of the gravitational field. Consequently, the gravity is now described by the nonmetricity of the spacetime. Indeed, the nonmetricity tensor **S** ($S^\lambda_{\mu\nu}$) describes the strength of the gravitational field in equation (14).

As we take a fresh look at our field equations[1] (14)-(15), we notice that the affine part of the Einstein tensor $\bar{G}_{\mu\nu} \equiv \bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}g_{\mu\nu}$ contains field potentials – metric tensor $g_{\mu\nu}$ – and, therefore, can hardly be justified as the field source as part of the stress-energy tensor. The only part that is independent of the gravitational field is the affine Ricci tensor $\bar{R}_{\mu\nu}$, which is calculated based on the affine connection $\bar{\Gamma}$ set *a priori*. Consequently, we shall modify our field equation as follows:

$$\hat{G}_{\mu\nu} - \frac{1}{2}\bar{R}g_{\mu\nu} = 8\pi\hat{T}_{\mu\nu}, \tag{16}$$

where the modified stress-energy tensor $\hat{T}_{\mu\nu}$ is defined as

$$\hat{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{8\pi}\bar{R}_{\mu\nu}. \tag{17}$$

It is easy to recognize in the field equation (16) the structure of Einstein's equation with a cosmological constant wherein the affine scalar curvature $\bar{R}$ plays the role of the cosmological constant Λ, although our "cosmological constant" is not necessarily constant. This curious similarity notwithstanding, the field equations (16)-(17) are identically equivalent to the classical Einstein field equations (13) *without* the cosmological constant.

The fact that the field equations (16)-(17) are equivalent to the standard equations of GR guarantees that there will be no tests that can distinguish between the two interpretations. It is then legitimate to ask, what are the advantages of this new geometrical interpretation of GR? The answer lies in the mathematical rigor and physical meaningfulness of the theory.

As has been known for a long time, GR suffers from certain difficulties related to the lack of general covariance of the theory. To wit, the energy-momentum pseudo-tensor of the gravitational field is not a covariant object, which leads to a lack of local conservation laws for the gravitational field – an unacceptable situation in our view. A popular attempt to explain away this difficulty by the principle of equivalence appears to be misguided. It fact, the principle of equivalence itself is not well defined in GR. This principle establishing local equivalence of the gravitational field and inertial forces arising in a noninertial FR, first of all, requires a good definition of the frame of reference, which, unfortunately, is all too often confused with a coordinate system. From here it is deduced that since gravity vanishes in a free-falling FR, there is nothing wrong with gravity vanishing in Riemannian coordinates, in which the Levi-Civita connection is zero. The fallacy of this argument is rooted in equating the Riemannian coordinate system with a free-falling FR. In this metric-affine reformulation of GR, the energy-momentum of the gravitational field is described by a covariant tensor.[1, 3] The global conservation laws also exist in this framework.[1, 3] Indeed, the Bianchi identity requires that

$$\begin{aligned} \nabla T &= 0, \\ \mathrm{T}^\mu_{\nu;\mu} &= 0. \end{aligned} \tag{18}$$

These conservation laws can be rewritten in our formalism as

$$\begin{aligned} \overline{\nabla}\,(T + t) &= 0, \\ \mathrm{T}^\mu_{\nu|\mu} + \mathrm{t}^\mu_{\nu|\mu} &= 0, \end{aligned} \tag{19}$$

where the semicolon denotes a covariant derivative with respect to Levi-Civita connection Γ, a vertical line denotes a covariant derivative with respect to affine connection $\bar{\Gamma}$, and t (t^μ_ν) is now a true tensor obeying covariant conservation laws.

Let us consider, for example, a trivial affine connection $\bar{\Gamma}$ with zero curvature. In this case, the Ricci tensor $\bar{R}_{\mu\nu}$ and the scalar curvature $\bar{R}$ of this affine connection $\bar{\Gamma}$ vanish. The nonmetric Einstein tensor $\hat{G}_{\mu\nu}$ differs from the regular Einstein tensor $G_{\mu\nu}$ only in that all partial derivatives of the metric are replaced with the covariant derivatives with respect to the affine connection $\bar{\Gamma}$: $\partial_\mu \to \delta_\mu^\lambda \bar{\nabla}_\nu = \delta_\mu^\lambda \partial_\nu - \bar{\Gamma}_{\mu\nu}^\lambda$. We see that the flat affine connection $\bar{\Gamma}$ plays here the role of a gauge field compensating for the arbitrary coordinate transformation and assuring general covariance of the theory.

To summarize, the mere existence of an auxiliary affine connection $\bar{\Gamma}$ allowed us to recast the Einstein field equations (without really changing them) in a form that suggests a novel geometrical interpretation of gravity as the nonmetricity of spacetime. This reformulation of General Relativity allows for ridding the theory of its difficulties related to noncovariance.

2 Geometrodynamics in a Frame of Reference

In the previous section we made a suggestion that the affine connection of the metric-affine space (L_4, g) may represent a frame of reference. In this section we will justify this hypothesis and show how the affine connection is determined in a chosen frame of reference. We will consider the concept of geometrodynamics as the guiding principle in describing any "universal"[4] force such as gravitational or inertial. Thus, the objective of this analysis is to find an appropriate geometric description of the noninertial frames of reference (NIFR) and the transformation laws between different frames of reference.

Einstein's GR, despite its claim to be the general theory of relativity, does not even define frames of reference. The principle of relativity is replaced by the principle of general covariance, confusing reference frames with coordinate systems, which play little role in the geometry of spacetime. This position is untenable because coordinate systems have no physical meaning whatsoever, while the frame of reference is a fundamental physical concept. A particular choice of a FR affects the physical laws therein.

As has been pointed out by Kretschman[5], Fock[6], Wigner[7], Rodichev[8], Mitzkevich[9] and a few other authors, the coordinate system is merely a way to number points or label events of spacetime.[10] Therefore, the general covariance principle is seen as devoid of physical meaning and a mere triviality.[11] We can well formulate both the geometry of spacetime and the physics in a given spacetime in the coordinate-free language of contemporary mathematics. It is for the purpose of illustrating this very point that we provided duplicative coordinate free representation for most of the above equations and geometrical objects.

There have been a number of attempts to describe frames of reference as chronometric invariants, monads or τ-fields and tetrads. We shall explore here another approach that stems directly from the very notion of geometrodynamics.

2.1 Special Relativity in an arbitrary coordinate system

As is well-known, according to the Special Theory of Relativity, the geometry in inertial frames of reference is the pseudo-Euclidean geometry of Minkowski four-dimensional space-time.

In special relativity it is accepted that (a) IFRs are represented by Lorentz Coordinate systems and that Lorentz transformations, which are representations of the Lorentz group of rotation in the Minkowski spacetime, describe the transition from one IFR to another. Generally speaking, coordinate systems, being merely a scheme of numbering points on a manifold (in our case, the events of the spacetime continuum), are devoid of any physical meaning and do not describe any reference frames, inertial or noninertial. Different IFRs are represented by different Minkowski spaces and the transition from one IFR to another is a diffeomorphism $\Re : M^1 \to M^2$.

A Minkowski space is a four-dimensional differential manifold whose points are space-time events, with a Minkowski metric η defined on the manifold. Assuming that different observers experience the same events albeit from different vantage points[†], we can assume that all events constitute a single manifold and different inertial observers correspond to different Minkowski metrics defined over the same differential manifold. Thus, the transition from one IFR to another in geometric terms amounts to the transformation $\eta^1 \rightarrow \eta^2$. More precisely, since, generally, the curvature is determined by the connection rather than by the metric, although Minkowski space has no curvature, nevertheless, it more correct to say that the transition from one IFR to another in geometric terms amounts to the transformation $\Gamma^1 \rightarrow \Gamma^{2}$[‡]. Due to the fact that Minkowski space, as a pseudo-Euclidian space, is flat, different Minkowski spaces, i.e. different Minkowski metrics on the manifold, are essentially identical up to a general coordinate transformation. Such coordinate transformations do not affect the metric, which is invariant, but they do affect the connection, which is not a covariant object. It is important to remember that an observer in an IFR is free to choose any coordinate system, which does not necessarily have to be a Lorentz (pseudo-Cartesian) coordinate system. Of course, in a flat space, such as Minkowski, it is always possible (and preferable) to select a global orthogonal pseudo-Cartesian coordinate system such as Lorentz, in which all connection coefficients vanish globally, as is frequently done; but this is an option, not a requirement.

These facts explain why associating IFR with Lorentz coordinate systems in Special Relativity, just as the use of Galilean coordinates to describe IFR in Newtonian mechanics (in a flat 3+1 Euclidian space), is acceptable for all practical purposes (albeit conceptually misleading) and does not lead to any contradictions. This situation changes radically as we attempt to describe noninertial frames of reference. A failure to recognize that a coordinate transformation, no matter how complex, can never describe a transition from an IFR to a NIFR or from one NIFR to another NIFR, has led to much confusion.

2.2   Geometrodynamics in a Noninertial frame of reference

Let us consider an observer traveling in a spacecraft. Suppose that from the point of view of another inertial observer, the spacecraft, which we will consider to be the reference body of the NIFR associated with the observer traveling therein, is accelerating with a constant acceleration $\boldsymbol{a}$ along a straight line in a 3-D space, which translates into a hyperbolic motion in the Minkowski 4-D spacetime of the inertial observer. If the observer in the spacecraft observes a few test particles freely moving inside the craft, she will notice that they all move with acceleration $-\boldsymbol{a}$ in the direction opposite of the direction of the spacecraft (as indicated by the accelerometers). If the test particles made of different material all accelerate uniformly, this suggests to the observer that either these particles move under the influence of a universal force (in Reichenbach's terminology) or that the spacetime is non-Euclidean. Thus, considering the motion of the test particles, the observer in the NIFR of the spacecraft has two choices:

1. to postulate the Minkowski spacetime and assume existence of a universal force, which acts upon these test particles and causes them all to accelerate with acceleration $-\boldsymbol{a}$, or

2. to rule out any universal forces and to admit that the geometry inside the spacecraft is *not* Euclidian (or rather not pseudo-Euclidian, i.e. not Minkowski), i.e. the spacetime is not flat.

[†]We are disregarding here the fact that an event visible to one observer may not be visible to another observer if this event lies outside his event horizon.

[‡]Needless to say, in Minkowski space the connection happens to be metric compatible, i.e. a flat Levi-Civita connection.

As Poincaré pointed out very early (and Reichenbach stressed later), the reality is the sum total of physics and geometry:

EMPIRICAL REALITY = PHYSICS + GEOMETRY

Although one is free to choose where the separation line between physics and geometry lies and, therefore, each of the two choices above are legitimate, Poincaré and Reichenbach advocated the second choice whereby all universal forces are eliminated. This is the principle of geometrodynamics.

Pursuant to the second choice, as dictated by geometrodynamics, the geometry of NIFR is non-Euclidean. The question remains, however, how to determine precisely the geometry within a NIFR.

To do that, let us first consider the movement of a spacecraft in an inertial frame of reference.

Let M be Minkowski spacetime with metric η and connection $\bar{\Gamma}$, which in a local chart x have respective components of $\eta_{\mu\nu}$ and $\bar{\Gamma}^\lambda_{\mu\nu}$. All freely moving test particles in the Minkowski space of an IFR obey the following equation:

$$\bar{\nabla}_X X = 0, \tag{20}$$

where $\bar{\nabla}$ is a covariant derivative with respect to the connection $\bar{\Gamma}$. In the local chart x the equation (20) takes the form:

$$\frac{d^2 x^\lambda}{d\tau^2} + \bar{\Gamma}^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0, \tag{21}$$

where τ is a smooth affine parameter along the worldline, which can be taken to represent the proper time of this test particle.

Expression (21) is the equation for a geodesic line in an arbitrary coordinate system x. Similarly, the equation for a reference body (in our example, the spacecraft) accelerating with acceleration a takes the form of

$$\bar{\nabla}_X X = a, \tag{22}$$

or

$$\frac{d^2 x^\lambda}{d\tau^2} + \bar{\Gamma}^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = a^\lambda. \tag{23}$$

Let us now consider the movement of the same test particles from within the spacecraft, i.e., from a NIFR. All of the test particles inside the spacecraft are accelerating with respect to an observer inside the craft with the same acceleration a but in the opposite direction:

$$\bar{\nabla}_X X = -a, \tag{24}$$

or

$$\frac{d^2 x^\lambda}{d\tau^2} + \bar{\Gamma}^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -a^\lambda. \tag{25}$$

If we do not insist on maintaining flat Minkowski geometry and do not wish to admit the existence of universal forces causing acceleration $-\boldsymbol{a}$, i.e. we choose a geometrodynamical representation of reality, we have to assume that these test particles move along the geodesic lines of a non-Euclidian space — a space of affine connection Γ:

$$\nabla_X X = 0, \tag{26}$$

or

$$\frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0, \tag{27}$$

where ∇ is a covariant derivative with respect to the affine connection Γ and $\Gamma^\lambda_{\mu\nu}$ are the components of the connection Γ. Eliminating the force acting on a test particle by describing its motion as a free fall along geodesics in a non-Euclidean space is the essence of geometrodynamics, which aims to describe the field of force as a manifestation of non-Euclidean geometry of spacetime.

Note that the equations (25) and (27) describe the same trajectory of the same test particle. (Since the affine parameter τ is related only to the curve representing the trajectory of a test particle, which is a geometrical invariant, we are justified in using the same affine parameter for (25) and (27) describing the same curve.) Hence, deducting (25) from (27) we obtain

$$T^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = a^\lambda, \tag{28}$$

where

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \bar{\Gamma}^\lambda_{\mu\nu} \tag{29}$$

is called the tensor of affine deformation. It is easy to see that the Minkowski metric η is inhomogeneous with respect to the affine connection Γ, i.e. its covariant derivative with respect to this connection does not vanish: $\nabla \eta \neq 0$. Generally, according to (1), the affine deformation $T^\lambda_{\mu\nu}$ is comprised of the symmetric tensor of nonmetricity $S^\lambda_{\mu\nu}$ and anti-symmetric torsion tensor $Q^\lambda_{\mu\nu}$:

$$T^\lambda_{\mu\nu} = S^\lambda_{\mu\nu} + Q^\lambda_{\mu\nu}. \tag{30}$$

As is known, torsion does not affect geodesics, i.e. two affine connections different only by torsion have the same geodesics. Thus, for non-rotating FRs, we can disregard torsion and assume that affine connection $\Gamma^\lambda_{\mu\nu}$ is symmetric in its two lower indices:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}. \tag{31}$$

and, therefore, the tensor of affine deformation is equal to the tensor of nonmetricity:

$$T^\lambda_{\mu\nu} = S^\lambda_{\mu\nu}. \tag{32}$$

The tensor of nonmetricity can be expressed through the covariant derivatives of metric as follows:

$$\eta_{\tau\sigma} S^\tau_{\mu\nu} = \frac{1}{2} \left(\nabla_\mu \eta_{\nu\sigma} + \nabla_\nu \eta_{\mu\sigma} - \nabla_\sigma \eta_{\mu\nu} \right). \tag{33}$$

Furthermore, it is interesting to note that any geodesic transformation of the affine connection, i.e. a transformation of the type:

$$\tilde{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu} + \frac{1}{2} \left(p_\mu \delta^\lambda_\nu + p_\nu \delta^\lambda_\mu \right), \tag{34}$$

where p_μ is an arbitrary covariant vector, does not affect the geodesics. Consequently, the geometry of a NIFR based on the worldline geodesics is defined only up to an arbitrary torsion and an arbitrary geodesic transformation of the type (34).

2.3 Geodesics or Autoparallel lines?

Since it turns out that the space in a NIFR is a space of affine connection with nonmetricity (and, possibly, torsion), we have to retrace our steps and take a closer look at the step leading to equations (26) and (27). We called the trajectories of the test particles geodesic lines. There is a certain inconsistency in this terminology that may lead to confusion. In geometry, the "straightest" line defined by the parallel transport of a tangent vector, i.e. by the affine connection, is always called a geodesic line. The shortest line or, more generally, the line of a stationary length, which requires a metric, is called the extremal path of a certain functional. In Riemannian space, where the Levi-Civita connection is metrically compatible, the straightest lines coincide with the lines of stationary length and both are called geodesics. In a space of affine connection these two types of curves no longer coincide. In physics literature, it has become accepted to call the "shortest" line, or rather, the line of extremal length, *geodesic*, and the straightest line, *auto-parallel*. The line with the extremal proper length is derived through a variational principle of Euler-Lagrange:

$$\delta s = \int\limits_A^B ds = \int\limits_A^B (-g_{\mu\nu}dx^\mu dx^\nu)^{1/2} = 0 \tag{35}$$

or

$$\frac{\delta I}{\delta x^\sigma} = \frac{1}{2} \int\limits_A^B g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda = 0 \tag{36}$$

The "straightest" line, on the other hand, is an expression of parallel transport defined by connection, merely requiring that a vector is transported parallel to itself along the line s, which is called an autoparallel line. Which line, the "straightest" or the "shortest," are we to take as an expression of the trajectory of a test particle?

In our view, the "shortest" line is not a local concept. It requires a line between two points A and B to have the shortest (or extremal) proper length (or time for a timelike worldline). A test particle in any given point on its trajectory "knows" nothing about the length of the curve between the point where the particle is and some other point where it *is not*. Consequently, it is illogical to assume that the freely moving test particle will follow the "shortest" line. Such an assumption would imply that the particle in a given point on the worldline somehow is aware of the global properties of this worldline extending into the future.

On the other hand, the "straightest" or autoparallel line is a local concept. In any given point on the line, the connection and curvature of the line are defined in that point. This curvature can be minimized (or extremized) in that point. Thus the straightest line is a logical choice for a test particle to follow. Henceforth, we shall continue to use only the "straightest," i.e. auto-parallel, lines to describe trajectories of free test particles, but will retain for them the term *geodesics* as it is accepted in the literature on differential geometry.

2.4 Geometry in Noninertial Frames of Reference

Let us now proceed to calculate the curvature of a NIFR. To achieve this goal we need to find the solution to equation (28), which we will rewrite here in a slightly different form:

$$T_{\mu\nu}^\lambda u^\mu u^\nu = a^\lambda, \tag{37}$$

where the local velocity of the test particle is denoted as $u^\mu = dx^\mu/dt$. Suppose $C_{\mu\nu}$ is a covariant tensor of the second rank. Let us consider projection of this tensor on the two velocity vectors u^μ and u^ν:

$$C_{\mu\nu}u^{\mu}u^{\nu} = c^2, \tag{38}$$

where c^2 is an invariant scalar, which we for simplicity shall consider a constant. Then a solution of the equation for affine deformation (37) takes the form:

$$T^{\lambda}_{\mu\nu} = \frac{1}{c^2}a^{\lambda}C_{\mu\nu}. \tag{39}$$

Whenever an affine connection $\bar{\Gamma}$ is transformed into a connection Γ by affine deformation $\mathbf{T}^{\S}$: $\Gamma=\bar{\Gamma}+\mathbf{T}$, or in components

$$\Gamma^{\lambda}_{\mu\nu} = \bar{\Gamma}^{\lambda}_{\nu\mu} + T^{\lambda}_{\nu\mu}, \tag{40}$$

the Riemannian curvature tensor $\mathbf{R}$ undergoes the following transformation:

$$R^{\sigma}_{\lambda\mu\nu} = \bar{R}^{\sigma}_{\lambda\mu\nu} + \bar{\nabla}_{\lambda}T^{\sigma}_{\mu\nu} - \bar{\nabla}_{\mu}T^{\sigma}_{\lambda\nu} + T^{\sigma}_{\lambda\rho}T^{\rho}_{\mu\nu} - T^{\sigma}_{\mu\rho}T^{\rho}_{\lambda\nu} + 2Q^{\rho}_{\lambda\mu}T^{\sigma}_{\rho\nu}, \tag{41}$$

where $R^{\sigma}_{\lambda\mu\nu}$ is the Riemannian curvature tensor of the second affine connection Γ, $\bar{R}^{\sigma}_{\lambda\mu\nu}$ is the Riemannian curvature tensor of the first connection $\bar{\Gamma}$ (in our case, this is the Levi-Civita connection of the Minkowski space in an IFR), $\bar{\nabla}$ is the covariant derivative with respect to the first connection $\bar{\Gamma}$, $\mathbf{T}$ ($T^{\lambda}_{\mu\nu}$) is the affine deformation and $\mathbf{Q}$ ($Q^{\lambda}_{\mu\nu}$) is the torsion of the second connection Γ.

Note that the Levi-Civita connection of the Minkowski space in IFR is flat – hence its curvature tensor is zero: $\bar{R}^{\sigma}_{\lambda\mu\nu} = 0$. Furthermore, since the torsion leaves geodesics invariant, we will for now disregard it: $Q^{\lambda}_{\mu\nu}=0$. Now expression (41) takes a simpler form of

$$R^{\sigma}_{\lambda\mu\nu} = \bar{\nabla}_{\lambda}T^{\sigma}_{\mu\nu} - \bar{\nabla}_{\mu}T^{\sigma}_{\lambda\nu} + T^{\sigma}_{\lambda\rho}T^{\rho}_{\mu\nu} - T^{\sigma}_{\mu\rho}T^{\rho}_{\lambda\nu} \tag{42}$$

or

$$R^{\sigma}_{\lambda\mu\nu} = \bar{\nabla}_{[\lambda}T^{\sigma}_{\mu]\nu} + T^{\sigma}_{\rho[\lambda}T^{\rho}_{\mu]\nu}. \tag{43}$$

Substituting in (42) the value derived for the affine deformation from (39), we get

$$R^{\sigma}_{\lambda\mu\nu} = \frac{1}{c^2}\left[\bar{\nabla}_{\lambda}\left(a^{\sigma}C_{\mu\nu}\right) - \bar{\nabla}_{\mu}\left(a^{\sigma}C_{\lambda\nu}\right)\right] + \frac{1}{c^4}a^{\sigma}a^{\rho}\left(C_{\lambda\rho}C_{\mu\nu} - C_{\mu\rho}C_{\lambda\nu}\right) \tag{44}$$

or

$$R^{\sigma}_{\lambda\mu\nu} = \frac{1}{c^2}\left(C_{\mu\nu}\bar{\nabla}_{\lambda}a^{\sigma} - C_{\lambda\nu}\bar{\nabla}_{\mu}a^{\sigma} + a^{\sigma}\bar{\nabla}_{[\lambda}C_{\mu]\nu}\right) + \frac{1}{c^4}a^{\sigma}a^{\rho}\left(C_{\lambda\rho}C_{\mu\nu} - C_{\mu\rho}C_{\lambda\nu}\right). \tag{45}$$

The Ricci curvature tensor defined as $R_{\mu\nu} = R^{\lambda}_{\lambda\mu\nu}$ takes the form

$$R^{\lambda}_{\lambda\mu\nu} = \frac{1}{c^2}\left[\bar{\nabla}_{\lambda}\left(a^{\lambda}C_{\mu\nu}\right) - \bar{\nabla}_{\mu}\left(a^{\lambda}C_{\lambda\nu}\right)\right] + \frac{1}{c^4}a^{\lambda}a^{\rho}\left(C_{\lambda\rho}C_{\mu\nu} - C_{\mu\rho}C_{\lambda\nu}\right) \tag{46}$$

or

$$R_{\mu\nu} = \frac{1}{c^2}\left(C_{\mu\nu}\bar{\nabla}_{\lambda}a^{\lambda} - C_{\lambda\nu}\bar{\nabla}_{\mu}a^{\lambda} + a^{\lambda}\bar{\nabla}_{[\lambda}C_{\mu]\nu}\right) + \frac{1}{c^4}a^{\lambda}a^{\rho}\left(C_{\lambda\rho}C_{\mu\nu} - C_{\mu\rho}C_{\lambda\nu}\right). \tag{47}$$

We can also calculate the scalar curvature by contracting the Ricci tensor $R_{\mu\nu}$ with the Minkowski metric $\eta^{\mu\nu}$:

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§ This (1,2) tensor of affine deformation T ($T^{\lambda}_{\mu\nu}$) should not be confused with the (0,2) stress-energy tensor T ($T_{\mu\nu}$) used in the field equations (13).

$$R = \frac{1}{c^2} \left(C \overline{\nabla}_\lambda a^\lambda - C_\lambda^\mu \overline{\nabla}_\mu a^\lambda + \eta^{\mu\nu} a^\lambda \overline{\nabla}_{[\lambda} C_{\mu]}^\mu \right) + \frac{1}{c^4} a^\lambda a^\rho \left(C_{\lambda\rho} C - C_{\mu\rho} C_\lambda^\mu \right), \qquad (48)$$

where $C = \eta^{\mu\nu} C_{\mu\nu}$.

Needless to say, the four equations (37) are not enough to uniquely define the tensor of affine deformation $T_{\mu\nu}^\lambda$, which has 40 components. We need an additional assumption to determine the connection. Let us now make some assumptions about the tensor $C_{\mu\nu}$. The simplest and the most important covariant tensor of the second rank that exists in our geometry is the Minkowski metric tensor $\eta_{\mu\nu}$. It is also symmetric, as is $C_{\mu\nu}$, and appears to be the most natural candidate for the role of $C_{\mu\nu}$. Consequently, we are going to assume that

$$C_{\mu\nu} = \eta_{\mu\nu}. \qquad (49)$$

The expression

$$T_{\mu\nu}^\lambda = \frac{1}{c^2} a^\lambda \eta_{\mu\nu} \qquad (50)$$

is certainly a solution of equation (37) and seems to be the most meaningful physically. This assumption allows us to rewrite the expressions for the curvature tensors as follows:

$$R_{\lambda\mu\nu}^\sigma = \frac{1}{c^2} \left(\eta_{\mu\nu} \bar{\nabla}_\lambda a^\sigma - \eta_{\lambda\nu} \bar{\nabla}_\mu a^\sigma \right) + \frac{1}{c^4} a^\sigma \left(a_\lambda \eta_{\mu\nu} - a_\mu \eta_{\lambda\nu} \right), \qquad (51)$$

$$R_{\mu\nu} = \frac{1}{c^2} \left(\eta_{\mu\nu} \bar{\nabla}_\lambda a^\lambda - \bar{\nabla}_\mu a_\nu \right) + \frac{1}{c^4} \left(a^2 \eta_{\mu\nu} - a_\mu a_\nu \right), \qquad (52)$$

$$R = \frac{3}{c^2} \left(\bar{\nabla}_\lambda a^\lambda + \frac{a^2}{c^2} \right). \qquad (53)$$

Let us suppose now that the NIFR is uniformly accelerating, i.e., the acceleration $\boldsymbol{a}$ is constant. That assumption allows us to further simplify the above expressions:

$$R_{\lambda\mu\nu}^\sigma = \frac{1}{c^4} a^\sigma \left(a_\lambda \eta_{\mu\nu} - a_\mu \eta_{\lambda\nu} \right), \qquad (54)$$

$$R_{\mu\nu} = \frac{1}{c^4} \left(a^2 \eta_{\mu\nu} - a_\mu a_\nu \right), \qquad (55)$$

$$R = \frac{3a^2}{c^4}. \qquad (56)$$

Although in geometric units, an assumption $C_{\mu\nu} = \eta_{\mu\nu}$ necessarily leads c=1, because the square of the proper velocity vector is a unity: $u^2 = u^\mu u_\mu = 1$, it is informative to consider the expression (56) in real physical units. It is easy to see that the constant c has physical units of velocity [m/sec] and, in fact, coincides with the speed of light. We see that although acceleration causes curvature of spacetime, this curvature is remarkably small – on the order of $1/c^4$.

3 General Relativity in a Noninertial frame of reference

The discussion in Part I, in which we analyzed Einstein's field equation in the presence of an arbitrary affine connection, has revealed that the gravitational field ought to be described as nonmetricity of spacetime. More specifically, the field equations would look like Einstein's standard equations wherein the partial derivatives of the metric tensor are replaced by covariant derivatives with respect to the affine connection. However, even in their modified form (14)-(15), Einstein's equations of GR are hardly suited to describe the gravitational

field in a noninertial frame of reference. Indeed, we started our analysis with the assumptions that the "ultimate" geometry is that of Riemann and that the test particles move along the geodesics of the Levi-Civita connection, which we merely decomposed into its affine and nonmetric components. The reality may not be that simple, and there is no reason to believe that the geometry in a noninertial frame of reference is described by a Levi-Civita connection. In fact, from the above analysis, we can easily see that it is not the case.

Let us consider the evolution of the geometry one step at a time. Let us start with the inertial frame of reference in the absence of the gravitational field. According to the special theory of relativity, the geometry in such a case is that of a Minkowski space with a flat metric and a compatible Levi-Civita connection $\overset{0}{\Gamma}$.

As the next step, let us consider a noninertial frame of reference. According to our analysis in Part II, the transformation from an IFR to a NIFR will subject our Levi-Civita connection to an affine deformation (29) resulting in a metric-affine space (L_4,g) with the Minkowski metric η and the independent affine connection Γ defined as

$$\overset{1}{\Gamma} = \overset{0}{\Gamma} + \overset{1}{T},\tag{57}$$

where the affine deformation $\overset{1}{T}$ is defined by (40).

Finally, let us introduce a gravitational field into this NIFR. As we concluded before, the gravity should be described as nonmetricity. Thus, our affine connection of NIFR $\overset{1}{\Gamma}$ undergoes another affine deformation

$$\overset{2}{\Gamma} = \overset{1}{\Gamma} + \overset{2}{\mathbf{T}},\tag{58}$$

where the affine deformation $\overset{2}{\mathbf{T}}$ is none other than a nonmetricity tensor $\mathbf{S}$ defined in (3), which is the strength of the gravitational field. Let us rewrite this expression in a more convenient format

$$\Gamma = \bar{\Gamma} + \mathbf{S},\tag{59}$$

where we have adapted the following notations: $\bar{\Gamma} = \overset{1}{\Gamma}$, $\Gamma = \overset{2}{\Gamma}$ and $\mathbf{S} = \overset{2}{\mathbf{T}}$. Consequently, the curvature tensors can be similarly decomposed as

$$\mathbf{R} = \bar{\mathbf{R}} + \hat{\mathbf{R}}\tag{60}$$

$$\mathbf{Rc} = \bar{\mathbf{Rc}} + \hat{\mathbf{Rc}}\tag{61}$$

$$R = \bar{R} + \hat{R}\tag{62}$$

$$\mathbf{G} = \bar{\mathbf{G}} + \hat{\mathbf{G}}\tag{63}$$

Einstein's equations neatly follow from the Bianchi identity, which requires that the covariant divergence of the Einstein tensor vanishes. The proportionality of the Einstein tensor $\mathbf{G}$ to the stress-energy tensor $\mathbf{T}$ assures, therefore, the conservation laws. We require that this proportionality of the Einstein tensor $\mathbf{G}$ to the stress-energy tensor $\mathbf{T}$ hold true in a metric affine space of a noninertial frame of reference, i.e. that the Einstein tensor $\mathbf{G}$ of the affine connection Γ is still proportional to the stress-energy tensor $\mathbf{T}$, which may now include the contribution of the inertial forces that have energy and, therefore, are an additional source of the gravitational field. It is only naturally to assume that such a contribution, i.e. the stress-energy tensor of the inertial forces, is equal to the Einstein tensor of the affine connection $\bar{\Gamma}$: $\mathbf{T}_{inert} = \bar{\mathbf{G}}$. Thus we have:

$$\bar{\mathbf{G}} + \hat{\mathbf{G}} = 8\pi\mathbf{T} + \bar{\mathbf{G}}. \tag{64}$$

Canceling the affine Einstein tensor on both sides of the equation, we arrive at the equations for the gravitational field in a noninertial frame of reference:

$$\begin{aligned} \hat{\mathbf{G}} &= 8\pi\mathbf{T}, \\ \hat{G}_{\mu\nu} &= 8\pi T_{\mu\nu.} \end{aligned} \tag{65}$$

As is easily seen, the equations (65) are Einstein's equations (13) in which all partial derivatives of the metric g are replaced by the covariant derivatives with respect to the affine connection $\bar{\Gamma}$.

$$\partial_\mu \to \delta_\mu^\lambda \bar{\nabla}_\nu = \delta_\mu^\lambda \partial_\nu - \bar{\Gamma}_{\mu\nu}^\lambda \tag{66}$$

These equations are no longer equivalent to Einstein's equations and represent a generalization of Einstein's theory for a gravitational field in a noninertial frame of reference. In the simplest case of the inertial frame of reference, the equations (65) are equivalent to Einstein's equation (13). Therefore, this generalization contains the classical GR in a special case of the gravitational field in an inertial frame of reference.

4   Conclusion

Analyzing GR in the presence of an arbitrary affine connection, we have determined that Einstein's equations describe the gravitational field as nonmetricity of an auxiliary affine connection. Since this result holds true for any affine connection, including a flat connection, we concluded that GR in fact describes gravity as nonmetricity of spacetime.

Analysis of the geometry in a noninertial frame of reference revealed that this is a metric-affine geometry and that the transformation between frames of reference is represented as an affine deformation.

It was concluded that GR, which demands Riemannian geometry, is inadequate to describe a gravitational field in a non-inertial frame of reference. A simple generalization of General Relativity we proposed has the field equations that revert to Einstein's equations in a special case of an inertial frame of reference.

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